

# FHFA STAFF WORKING PAPER SERIES



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September 2016

Working Paper 16-04

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Please address correspondences to William Doerner ([william.doerner@fhfa.gov](mailto:william.doerner@fhfa.gov)). Many thanks to Andy Leventis, Scott Smith, and Fred Graham, who provided helpful questions, comments, and discussion. We also thank discussants and participants from the George Washington University's University Seminar on Forecasting, the American Real Estate Society annual meetings, and the AREUEA national conference. No outside funding was received for this research.

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# Missing the Mark: House Price Index Accuracy and Mortgage Credit Modeling

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## Abstract

We make two contributions to the study of house price index and mortgage credit modeling accuracy. First, we assess the predictive power of house price indices calculated at different levels of geographic aggregation. Lower levels of aggregation offer superior fit when appreciation rates vary substantially across submarkets and the indices are based on a sufficient number of transactions. Second, we estimate a competing options credit model using 15 years of mortgage performance data in the United States. Model accuracy is highest when using indices at a city or lower level of aggregation to construct current loan-to-value ratios. Fit is weaker when using state or national price indices. Overall, this research highlights the benefits of using more localized house price indices when predicting property values and mortgage performance.

**Keywords:** house prices · loan-to-value · mortgage performance · credit model

**JEL Classification:** C55 · G22 · R30 · R51

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## 1. Introduction

Realtors have long heralded the claim that the residential housing market is driven by “location, location, location.” Nonetheless, a variety of limitations have prevented researchers from broadly testing the assertion that appreciation rates are dependent on a house’s particular site within a city. In finance, a better understanding of local house price dynamics could lead to a more accurate valuation of mortgage collateral and improved estimates of the credit risk associated with mortgage securities.<sup>1</sup> We attempt to determine the extent to which house price indices of different levels of granularity affect pricing accuracy, and the consequences of using different price indices on efforts to model mortgage prepayments and defaults.

A portfolio of mortgages can be a complex financial product to price. The value of the underlying collateral is likely to change as a result of both longitudinal and cross-sectional variation in house price appreciation rates. Urban real estate theory suggests that, even within the same city, housing can appreciate at different rates (see classic studies by Bailey et al., 1963; Alonso, 1964; Mills, 1967; Muth, 1969) for predictable reasons. But, up until now, such within-city cross-sectional variation has been difficult to measure reliably, stymieing research on the portfolio effects and credit risk implications of highly localized variation of house price appreciation.

We make two main contributions to the study of house price valuation and mortgage credit modeling. First, we assess the accuracy of house price indices (HPIs) calculated at different levels of geographic aggregation from a database of nearly 100 million observations. The indices allow us to predict sales values for a hold-out sample of individual properties and judge accuracy based upon subsequent market transactions. In general, differences in predictive accuracy occur when house price appreciation rates across sub-aggregates are different, and the increased signal from a more granular index is not counterbalanced by increased variance. Aggregate HPIs are better predictors in rural areas or small cities, and during periods of stable appreciation. Disaggregated indices produce more accurate estimates within large cities, especially during booms and busts in housing markets when substantial submarket appreciation differentials exist. Overall, we find the ideal level of house price index aggre-

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<sup>1</sup>We are not the only ones to suggest that granular information could be advantageous in mortgage markets. Recently, Stroebel (2016) studies how knowing a builder’s construction quality allows certain lenders to offer lower rates and incur less severe losses through superior loan modeling.

gation in large cities is at the 5-digit ZIP code level. In small cities, a city-level index is sufficient. We also find that a linear combination of indices at a variety of different levels of aggregation may offer superior fit.

Second, we use the HPIs to estimate mortgage performance across the United States for the last 15 years. A primary risk factor affecting the incidence of prepayment and default is a mortgage’s loan-to-value (LTV) ratio. In each period post-origination, the current LTV (CLTV) ratio is recalculated to account for principal payments and the marked-to-market value of the housing asset. Throughout the life of the loan, the CLTV is key to estimating a borrower’s likelihood to prepay or default. To understand the full effect of potential geographic aggregation bias, we begin by estimating a credit model where CLTV has been constructed using a national index. Next, we recalculate CLTV ratios with increasingly granular indices and show that using more localized HPIs generally improves model fit, particularly in central areas of larger cities. We also uncover some interesting cases in which increased granularity has some beneficial model impact at an individual asset level (e.g., to better identify underwater borrowers for modifications, refinancing, or workout options) as well as for portfolio analysis (e.g., to reduce model error in loss estimation).

The paper is structured as follows. Section 2 introduces a suite of HPIs constructed at different levels of geography and analyzes their predictive accuracy. We run several types of tests to investigate when and where an HPI is likely to “miss the mark.” Section 3 utilizes the same indices to evaluate the effect of geographic aggregation bias on mortgage credit modeling. Section 4 offers concluding thoughts.

## **2. Assessing House Price Index Accuracy**

House price indices can be useful for establishing an estimate of market values for individual properties—the collateral for loans in mortgage-backed securities. The suitability of an index depends on its predictive accuracy and the extent of its estimation error. Aggregation bias can occur when house prices appreciate at different rates in different submarkets, and an index constructed over several such submarkets masks those differences. Mitigating aggregation bias therefore requires estimating price indices at a more local level. Estimation error is a function of the variance of an estimator, which is affected primarily in this context by the number of transactions over which the index is estimated. The tension in the selection of a house price index is therefore a tradeoff between reduced aggregation bias and higher es-

timation error as index granularity increases. In this section, we investigate the appropriate HPI aggregation level to minimize these two separate sources of error.

To foreshadow, we find that an HPI’s accuracy fluctuates over time. It tends to become less precise as housing prices become more volatile (i.e., when prices are quickly falling or rising). We also find that accuracy improves as we move from a national index to increasingly disaggregated indices, although the incremental benefit from disaggregation decreases.<sup>2</sup> As we move to more and more local geographies, increased precision is overwhelmed by increased estimator variance. Indices using a ZIP code level of aggregation generally minimize the aggregate error from these competing sources, but less granular indices can perform just as well in certain contexts. For instance, in small cities or in periods when the housing market is growing, a city-level index will do at least as well. Next, we describe the construction of our house price indices, evaluation methods, and results.

## 2.1. House Price Measurement

We use a repeat-sales methodology to construct house price indices. A standard approach was introduced by Bailey, Muth, and Nourse (1963) and updated by Case and Shiller (1987). The method is attractive because of its limited data requirements (only requiring a price, transaction date, and location) and a straightforward interpretation. When a housing unit resells and its characteristics do not change, any gains in value reflect a change in house price rather than a change in quantity. Formally, we express a house’s value as a linear combination of unit-specific characteristics  $X$  with implicit prices  $\beta$  and a price level  $\delta$ , or

$$\ln(P_{ijt}) = X'_{ijt}\beta + D'_{jt}\delta_j + \epsilon_{ijt} \quad (1)$$

and where we assume constant-quality characteristics,  $E[X_{ijs}] = X_{ijt}$ , for a house  $i$  in a location  $j$  during a time period  $t$  or  $s$ .<sup>3</sup> For ease of explanation, we define the differencing between repeat-sales as  $p_{ij\tau} = \Delta \ln(P_{ij\tau}) \equiv \ln(P_{ijt}) - \ln(P_{ijs})$  where a first sale happens in period  $s$  and a second sale in period  $t$ . Likewise, we simplify other terms as  $d'_{j\tau} = \Delta D'_{j\tau}$ ,

<sup>2</sup>This is consistent with recent work by Andersson and Mayock (2014) who find LTV ratios suffer from measurement error when HPIs are constructed at coarse geographies, which negatively affects the precision of loss estimates.

<sup>3</sup>A value index can be used as a price index if we assume attributes are identical across all units over time, or  $E[X_{ij\tau}] = X_i$ .

and  $\omega_{ij\tau} = \Delta\epsilon_{ij\tau}$ . We perform a three-step weighted least squares estimation procedure as

$$p_{ij\tau} = d'_{j\tau}\delta_j + \omega_{ij\tau} \quad (2)$$

$$\omega_{ij\tau}^2 = \alpha_1 + \alpha_2(t - s) + \alpha_3(t - s)^2 + e_{ij\tau} \quad (3)$$

$$\frac{p_{ij\tau}}{\sqrt{\hat{\omega}_{ij\tau}^2}} = \left( \frac{d_{j\tau}}{\sqrt{\hat{\omega}_{ij\tau}^2}} \right) \delta_j + \frac{\omega_{ij\tau}}{\sqrt{\hat{\omega}_{ij\tau}^2}} \quad (4)$$

where we first estimate Equation 2, regress the residual squared deviations on the time between the two sales with Equation 3, and apply the predicted squared deviations from the second stage to the variables in the first stage to estimate Equation 4. The exponential of  $\hat{\delta}$  can be normalized to form indices that convey relative house price levels.<sup>4</sup>

We construct a suite of annual house price indices that represent a variety of geographic aggregation levels.<sup>5</sup> We begin with the dataset created by Bogin, Doerner, and Larson (2016) that offers HPIs for CBSAs, counties, ZIP3, and ZIP5 indices.<sup>6</sup> Additional HPIs are created for national, state, census tract, and census block group geographies. The underlying raw database encompasses 97 million mortgages purchased by Fannie Mae and Freddie Mac between 1975 and 2015.<sup>7</sup> When we implement the three-step procedure as outlined above, we apply two filters. We remove any pair of transactions with annualized appreciation rates

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<sup>4</sup>For the most part, we follow methodology used by the Federal Housing Finance Agency (FHFA). We differ by not restricting the constant term when performing the three-step estimation, using an annualized appreciation rate filter, removing repeat sales within the same year, and relaxing the requirement of when an index is reported. These differences are explained below. Although minor, the choices could lead to some differences with official FHFA indices if they were to be constructed on an annual basis (monthly and quarterly frequencies are provided currently on the agency’s website).

<sup>5</sup>HPIs could be created using other comparison groups. Examples include new construction sales with existing housing stock, high tiered versus low tiered price distributions, or owner-occupied versus investor properties. Repeat-sales (or constant quality) indices, though, are most commonly used to track appreciations across different geographic levels. For instance, Case-Shiller offers 10-city and 20-city “composites” while the FHFA compares a national index with census divisions each month. This paper focuses on annual indices, which means we forgo some temporal detail but we are able to construct HPIs at finer geographic levels.

<sup>6</sup>The label “3-digit” ZIP (ZIP3) codes refer to the first 3 numbers of the postal code. For example, the ZIP code (ZIP5) of 32308 would belong to the ZIP3 of 323. Historically, the first digit usually identified a group of states and the next two digits represented a subregion within the group. The label “CBSA” stands for Core Based Statistical Area, which includes both metropolitan statistical areas and micropolitan statistical areas and is defined by the Office of Management and Budget using data from the Census Bureau.

<sup>7</sup>To our best knowledge, this is the largest and most comprehensive historical mortgage database available.

Table 1: Observation Counts for House Price Indices

	Total Indexes	Number of Indexes Starting Prior to 2000	Average Number of Pairs per Index
National	1	1	48,776,216
State	51	51	956,396
ZIP3	879	868	55,490
CBSA	959	954	50,861
County	2,704	2,396	18,031
ZIP5	17,936	14,739	2,699
Census Tract	54,613	45,972	878
Census Block Group	112,636	83,350	395

**Note:** Numerical values reflect the number of indices available for each level of geography. The District of Columbia is included in the state HPIs. “Total” gives the total index count, irrespective of the start date of the index. “Pre-2000 Start” conveys the total index count for indices that begin in 2000 or before. “Pairs per index” is the total number of transaction pairs (after the filters and index cutoffs) divided by the total index count.

greater than +/-40% and when a property sells twice within a year.<sup>8</sup> An index is first reported when an area reaches 25 half-pairs in a single year and 100 cumulative paired sales.<sup>9</sup> A value is not reported in a particular year if an area has fewer than five half-pairs. The counts and sample coverage for this suite of indices are shown in Table 1.

To analyze house price accuracy over our eight levels of geographic aggregation we look at relative performance using two standard methods: the root-mean-square error (RMSE) of predicted prices and a series of encompassing tests. Both metrics are computed via the following procedure. We select 80% of transactions within a particular area and create “trial” price indices. The remaining 20% of transactions represent a hold-out sample. We identify houses with a subsequent sale to serve as a benchmark for actual changes in market value, independent of our estimation sample. Based upon an initial sale value  $V$  in time  $t$ , we use our trial indices to calculate implied appreciation and predict the subsequent sales value in time  $t + h$  as,

$$\hat{V}_{t+h} = V_t \times \frac{\hat{P}_{t+h}}{\hat{P}_t}. \quad (5)$$

<sup>8</sup>The appreciation rate filter is an annual average log-difference of 0.3, which coincides with the FHFA’s filter of 40% per year. The property turnover filter is necessary due to the annual frequency of indices. We do not implement further filters, adjustments, or screens.

<sup>9</sup>A “half-pair” is an alternative count measure for a repeat-sales index. Instead of counting paired properties, it is the transaction count where either the first or the second sale occurs in the given period.

The hold-out sample permits us to compare each index using information separate from the sample used to create the trial indices. Accuracy is calculated across aggregation levels with emphasis placed on variation between locations.<sup>10,11</sup> The next two subsections discuss two comparison methods and the results.

## 2.2. RMSE Prediction Errors

We begin with a root-mean-square prediction error, where  $\hat{e}_{n,t+h}$  is calculated as  $\hat{V}_{n,t+h} - V_{n,t+h}$  with a  $n$  indicating individual property pairs.<sup>12</sup> Note that this value is neither a forecast nor a residual.  $\hat{V}$  is computed with HPIS that include information during and after time  $t + h$ , and is not directly estimated during HPI construction.<sup>13</sup> The root-mean-square error is defined as

$$RMSE = \sqrt{\left(\frac{1}{N} \sum_{n=1}^N \hat{e}_{n,t+h}^2\right)} \quad (6)$$

and is the preferred evaluation metric in the forecast evaluation literature and specific investigations of house prices, such as Nagaraja, Brown, and Wachter (2014). The RMSE also lends itself to the construction of Theil's (1966)  $U$  statistic, which measures the ratio of the accuracy of two forecasts,  $U = (RMSE_1/RMSE_2)$ . Under the null hypothesis that the forecast accuracy is equal, the square of the  $U$  statistic follows an  $F$ -distribution with  $(N_1 - 1, N_2 - 1)$  degrees of freedom.

Figure 1 presents three graphs that gauging the predictive performance of our suite of HPIS over several dimensions, including mortgage holding period, year, and geography. Each graph is explained below.

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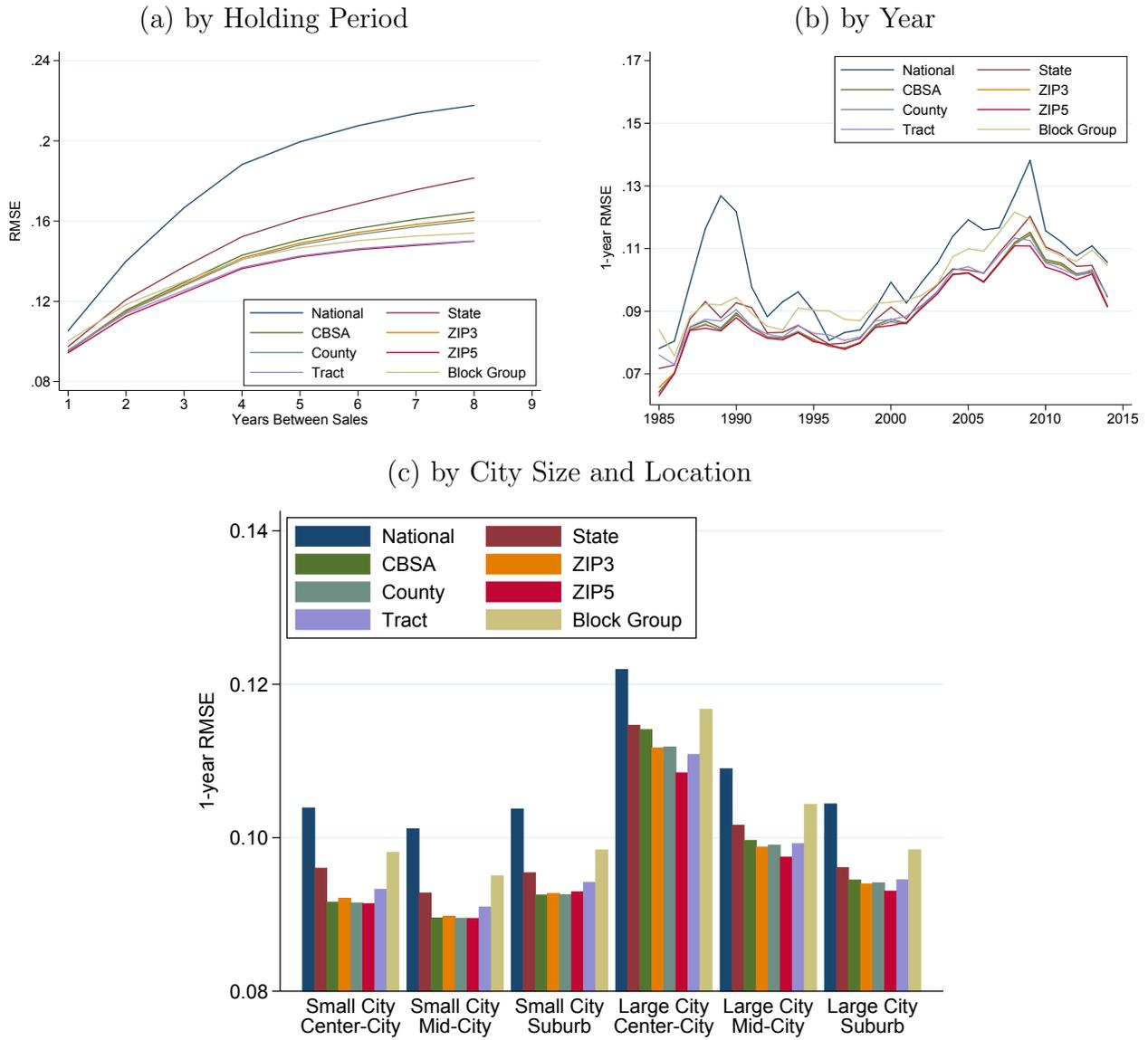
<sup>10</sup>Implicit is the assumption that the best indicator of a property's market value is the observed sale. We recognize that binding borrowing constraints, short time-on-the-market, and bidding wars may weaken this claim. Unfortunately, our analysis is constrained by the underlying data which include a rich set of transactions but limited information about each sale.

<sup>11</sup>We only produce accuracy metrics when an index exists for all eight levels of HPI aggregation.

<sup>12</sup>Recall that the mean-square error is the summation of the estimator's bias and variance. We expect the first term to be greater across larger areas (where estimates of individual properties might be measured with less precision because of the greater degree of appreciation rates) and the second term to be greater in smaller cities (where fewer observations lead to more noise and volatility in estimates).

<sup>13</sup>Instead, during index construction, *differenced* residuals are estimated.

Figure 1: Out-of-Sample Prediction Errors



The second stage of our HPI estimations (Equation 3) assumes that prediction errors can vary by the length of time between sales.<sup>14</sup> Panel (a) of Figure 1 shows that errors in price estimation increase at a decreasing rate as the holding period extends. This relationship holds across all HPI aggregation levels. Although the differences between the indices are small, they are statistically significant, with the ZIP5 index outperforming the other indices

<sup>14</sup>The holding period is the floor of the number of years between consecutive sales of the same property.

across every time period.<sup>15</sup> Interestingly, while the tract and block group indices are among the poorer performing local indices at a 1-year horizon, their relative ranking improves with time. National and state indices have the largest prediction errors as the holding period grows. This figure suggests that estimation error is not invariant to horizon length, and aggregation bias becomes pernicious with time. Another takeaway is that one should utilize a local submarket index for long-horizon price predictions.

Next, we examine prediction errors by calendar year. As illustrated in panel (b), prediction errors appear to be high during periods of declining house prices, from 2008 to 2010 for example, and errors tend to be smaller, regardless of the level of aggregation, when prices rise. A consequence of this result is that, *conditional* on having an accurate predicted price path, the *variability* of house prices rises as the index falls. This has an important potential implication for housing finance. Because mortgage defaults are a function of CLTVs, and CLTVs have a convex relationship with defaults, an increasing spread in house price appreciation rates could cause greater levels of default, conditional on price declines. In terms of aggregation, ZIP5 and county HPIs generally have the lowest error over time. *F*-tests with the null of equality between the ZIP5 index and the county index cannot be rejected at the 5% level except in three years (2009, 2010, and 2011). Equality between the ZIP5 and tract index is rejected in all but two years since 1992. Based upon this temporal comparison of minimum RMSE, the ZIP5 and county indices appear to be superior.

Finally, prediction errors might vary across geographies. In panel (c), when comparing the first three and second three bar graphs, we find that RMSE is lower in smaller cities.<sup>16</sup> The bar graphs illustrate how RMSE varies across different regions of a city. Regions are defined by proximity to the downtown central business district (CBD). Respectively, the regions represent concentric circles where the center-city is 0 to 5 miles from the CBD, the mid-

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<sup>15</sup>*F*-tests with the null of equality between the ZIP5 index and the next best index are rejected at the 0.01 level of significance for all holding periods (from 1.4 million observations in Year = 1 down to 350,000 in Year = 8).

<sup>16</sup>Small cities are defined as CBSAs with populations below 500,000 people in 1990. For expository purposes, we focus on subsequent sales that happen within the next year.

city is between 5 and 15 miles, and the suburbs are 15 to 25 miles from the downtown.<sup>17</sup> The bar graphs show substantial variation in errors. Small cities exhibit lower prediction errors than large cities, no matter which level of HPI granularity is used. However, the overall error-minimizing aggregation level is unclear; the U-shape (of the bar heights) is minimized for small cities at several aggregation levels, with a statistical tie between the CBSA, ZIP3, county, and ZIP5 indices. This finding is similar across each of the intra-city regions.<sup>18</sup> In stark contrast, large cities show greater deviations in RMSE across the index aggregation levels and regions within the city. Accuracy increases as we move to lower levels of aggregation until we reach the ZIP5 level, at which point the RMSE increases with HPI disaggregation.<sup>19</sup> The downward gradient across the regions (moving from the center-city out to the suburbs) tells us that large cities have greater prediction errors the closer a house is to the CBD.

A major implication from Figure 1 is that the ideal index—the one that minimizes prediction errors—is not the same for each area. In most locations, prediction errors can be reduced by moving to at least a CBSA or county level index. However, near the centers of large cities, a ZIP5 index further improves valuation accuracy. To the extent possible, a mixture of indices might be advisable when measuring CLTVs in a portfolio of loans. In the next subsection, we examine the predictive accuracy of a combination of indices.

### 2.3. Encompassing Tests

Encompassing tests provide another means of comparing the accuracy of our suite of HPIs. Instead of focusing on a singular index, we consider whether statistical gains might be achieved by modeling predictions with multiple aggregation index levels. An index’s prediction could be dominated by other aggregation levels, in terms of RMSE, yet have unique and relevant information that is useful for a prediction. For these tests, we follow the forecast

<sup>17</sup>The CBD is calculated as the maximum value within the CBSA of the inverse of the standardized land area plus the standardized share of housing units in 20+ unit structures. Land area data are from the Census’ TIGER line shapefiles, and structure type is from the 1990 Decennial Census, the earliest census for which ZIP code data are available. Distance to the CBD is calculated as the “as the crow flies” or straight-line distance. We categorize an area, like a ZIP code, as belonging to a particular region if its centroid is located in that concentric circle.

<sup>18</sup> $F$ -tests with the null of equality between the ZIP5 index and the CBSA index cannot be rejected at any reasonable level of significance for center-city, mid-city, and suburban areas within small cities ( $N = 90,000$ ,  $217,000$ , and  $139,000$ , respectively).

<sup>19</sup>The null of equality between the county and ZIP5 indices is rejected at the 0.1%, 0.1%, and 0.5% level of significance for all center-city, mid-city, and suburban areas within large cities ( $N = 31,000$ ,  $239,000$ , and  $541,000$ , respectively).

encompassing literature of Chong and Hendry (1986) and Ericsson and Marquez (1993). A predicted outcome is modeled as a function of a sequence of rival forecasts,  $i \in 1, 2, \dots, I$ , at horizon  $h$ , such that

$$V_{n,t+h} = \alpha_0 + \sum_{i=1}^I \alpha_i \hat{V}_{i,n,t+h} + e_{n,t+h} \quad (7)$$

where  $n$  indicates location and  $i$  denotes an index from our suite of geographic aggregation levels. For interpretation, the constant term,  $\alpha_0$ , controls for heterogeneity not captured by the forecasts and de-means the other variables. Parameters  $\alpha_1, \dots, \alpha_I$  indicate the relative importance of each aggregation level where the information content is conveyed by the sign and magnitude of the estimated coefficient. If the parameter is not statistically different than zero, the prediction is “encompassed.” Positive and significant estimates indicate the presence of unique and relevant information in the forecast. On the other hand, negative and significant estimates indicate the forecast may be highly collinear with another forecast and encompassed if the respective index is omitted.<sup>20</sup> Each estimated parameter denotes the fractional amount that it contributes to the prediction. By design, the summation of the statistically significant and positive parameter estimates should be close to one in a well-specified model.

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<sup>20</sup>Generally, the omission of the forecast with the negative coefficient should result in either one or more coefficients decreasing in an equal aggregate magnitude.

Table 2: Index Encompassing Tests

Model Sample	LHS Variable: <i>log</i> House Price						
	(1) Full	(2) Rural	(3) Small Cities	(4) Large Cities Center-City	(5) Large Cities Suburbs	(6) National HPI Growth	(7) National HPI Decline
National	0.018* [0.011]	-0.127*** [0.020]	-0.010*** [0.003]	0.066*** [0.014]	0.031*** [0.002]	-0.069*** [0.002]	<b>0.155***</b> <b>[0.004]</b>
State	0.001 [0.022]	0.078* [0.045]	-0.012* [0.007]	0.044 [0.037]	0.025*** [0.006]	0.016*** [0.005]	0.028*** [0.007]
CBSA	-0.004 [0.023]	<b>0.403***</b> <b>[0.053]</b>	<b>0.189***</b> <b>[0.025]</b>	-0.271*** [0.057]	-0.029*** [0.011]	0.005 [0.012]	-0.006 [0.015]
ZIP3	0.145*** [0.018]	0.072* [0.042]	0.114*** [0.013]	0.148* [0.088]	0.138*** [0.010]	<b>0.193***</b> <b>[0.010]</b>	0.071*** [0.012]
County	0.103*** [0.022]	<b>0.402***</b> <b>[0.055]</b>	0.099*** [0.025]	<b>0.285***</b> <b>[0.095]</b>	0.072*** [0.012]	<b>0.205***</b> <b>[0.013]</b>	0.005 [0.016]
ZIP5	<b>0.461***</b> <b>[0.012]</b>	0.110*** [0.036]	<b>0.366***</b> <b>[0.010]</b>	<b>0.458***</b> <b>[0.036]</b>	<b>0.483***</b> <b>[0.007]</b>	<b>0.393***</b> <b>[0.007]</b>	<b>0.465***</b> <b>[0.009]</b>
Census Tract	<b>0.195***</b> <b>[0.007]</b>	-0.024 [0.038]	<b>0.177***</b> <b>[0.009]</b>	<b>0.152***</b> <b>[0.030]</b>	<b>0.198***</b> <b>[0.006]</b>	<b>0.168***</b> <b>[0.006]</b>	<b>0.210***</b> <b>[0.009]</b>
Census Block Group	0.040*** [0.004]	0.026 [0.020]	0.036*** [0.005]	0.051*** [0.018]	0.043*** [0.004]	0.036*** [0.004]	0.041*** [0.005]
Constant	0.522*** [0.014]	0.760*** [0.022]	0.530*** [0.004]	0.868*** [0.019]	0.530*** [0.003]	0.671*** [0.003]	0.407*** [0.004]
Observations	1,494,788	24,664	486,557	34,048	974,183	1,096,700	398,088
R-squared	0.945	0.919	0.94	0.925	0.945	0.935	0.959

**Note:** \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ . The unit of observation is a single housing unit transaction. The log of the transaction price is modeled as a function of each index-based estimate. Standard errors are adjusted based on clustering by year  $\times$  CBSA code. **Bold** values indicate a statistically significant point estimate greater than 0.15.

We start with the unit of observation being a single housing unit transaction. As with the RMSE evaluation methods, an initial sale establishes the base price and an index is used to predict the subsequent sale price of the same unit. This market prediction is used in Equation 7. Results of index encompassing tests are presented in Table 2 where the actual value of a housing unit is regressed on a linear function of a constant term and a vector of forecasts from our suite of HPIs. Each column represents a different sample of housing transactions to test when the combined information content might be more or less important.

Column 1 gives the full sample of transactions (excluding the first transaction which is used as the base for the predictions) in the hold-out sample of nearly 1.5 million transactions where the holding period is 1 year. The ZIP3, county, ZIP5, census tract, and census block group indices each provide unique and relevant information to the prediction of house prices. The two most important HPIs (ZIP5 and census tract) are highlighted in bold. A single

index's estimate does not dominate the others. Rather, similar to Granger and Ramanathan (1984), a weighted average or a composite index may best predict the price path of individual housing units.

Columns 2 to 5 depict results for a series of subsamples divided by city size and location. In rural areas, indices with larger levels of geographic aggregation are preferred.<sup>21</sup> The CBSA and county HPIs are the most relevant, with the ZIP3 and ZIP5 indices contributing less significantly. Small cities offer a story that begins to emphasize disaggregate indices. As we move across the columns and city size increases, proximity to the downtown becomes more important, a result echoed in Bogin, Doerner, and Larson (2016). The ZIP5 HPI is most influential while census tract and census block group measures contribute additional information. In general, granular indices are more important for denser areas. More variation in density amplifies the variation in appreciation rates, which increases the prevalence of bias when considering larger levels of geographic aggregation.

Finally, in the last two columns, we consider the performance of indices when the national index is increasing versus decreasing. Column 6 shows that when house prices are rising, a variety of indices contribute to the optimal composite, with ZIP3, county, ZIP5 and census tract indices receiving the largest weights. However, when prices fall, the national, ZIP5, and census tract HPIs exhibit the most explanatory power. The two columns suggest local factors can drive booms and busts but the national trend becomes an important explanatory aspect of declines, perhaps because worsening macroeconomic conditions, like financial conditions and unemployment, are felt everywhere.

Overall, the encompassing tests present a nuanced view of house price index usage. No single index emerges as the dominate measure. Rather, a weighted average, depending on the area and condition of the national housing market, could give a superior predictive index. County, ZIP5, and census tract indices provide the most consistently high levels of predictive power.<sup>22</sup> Since submarket indices were also found to be important in the RMSE evaluations, it seems reasonable that mortgage performance modeling might be improved by using local HPIs to value houses that serve as collateral. We investigate this conjecture in the next section.

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<sup>21</sup>The "rural" sample is defined as housing units not contained within a CBSA.

<sup>22</sup>While they may be useful in other applications, state-level indices do not appear to provide information that is not already picked up by the other indices.

### 3. Modeling Mortgage Performance

Mortgage performance is often modeled in the context of a competing-options framework where a borrower makes mutually exclusive choices to continue with timely mortgage payments, prepay a mortgage, or default on the loan (Lawrence and Arshadi, 1995; Deng, Quigley, and Van Order, 2000; Calhoun and Deng, 2002; Dunskey and Pennington-Cross, 2003; Ergunor and Moulton, 2014). One of the key variables driving borrower behavior in these models is a mortgage’s loan-to-value (LTV) ratio. Empirical studies find that higher LTV ratios increase the probability of mortgage default (Schwartz and Torous, 1993; Sherlund, 2008; Mayer, Pence, and Sherlund, 2009; Pennington-Cross and Ho, 2010; Bacon and Moffatt, 2012; Fuster and Willen, 2013; Lam, Dunksy, and Kelly, 2013).<sup>23</sup> LTV ratios exceeding one indicate negative equity, where the borrower is underwater or owes more than the value of the underlying housing asset. Negative equity can lead homeowners to significantly cut back on improvements as well as mortgage principal payments (Olney, 1999; Melzer, 2013) and substantial evidence indicates borrowers strategically default when houses go underwater (Deng, Quigley, and Van Order, 2000; Bajari, Chu, and Park, 2008; Bhutta, Dokko, and Shan, 2010; Ghent and Kudlyak, 2011; Guiso, Sapienza, and Zingales, 2013; Chan, Haughwout, Hayashi, and van der Klaauw). All of these studies focus on the LTV ratio, which is composed of two parts—loan amount and house value—both crucial to the construction of an accurate measure. Loan amount is simple to establish with an actual payment history or, when unavailable, an amortization table, but market value is not so easily estimated.

Market value can be estimated in a variety of ways. The LTV at origination is usually based on a house’s appraisal or sales value. Afterwards, for ongoing analyses, the LTV is updated with an estimate of current market value, generally derived from observed comparable sales or statistical modeling.<sup>24</sup> The former involves individual expertise, human judgement, and a forecasting of local market trends but requires considerable time and resources; the latter uses automated techniques to generate quick and inexpensive estimates but the quality of the results is largely determined by the richness and accuracy of the underlying property

<sup>23</sup>Studies find this result with both LTV measured at origination and updated to a current period. We focus our attention on latter to examine the combined effect of LTV at origination, principal and interest payments, and local house price appreciation on the probability of prepayment and default.

<sup>24</sup>The new ratio is known commonly as the current LTV (CLTV) or marked-to-market LTV (MTMLTV). Although perhaps confusing, the CLTV acronym is sometimes used as a distinct but related shorthand for a combined loan-to-value ratio where a second lien is included in the loan value to measure total leverage.

transactions data. These automated techniques often involve an HPI. Our earlier findings demonstrate that HPI aggregation can have a meaningful effect on accurate property valuation. Now, we take that one step further by examining whether different levels of accuracy affect mortgage performance modeling.

We begin this analysis by constructing a model using the variables and techniques commonly applied in the mortgage finance literature. To do this, we draw from a nationwide sample of single-family mortgage originations that tracks borrower performance behavior between 1999 and 2014.<sup>25</sup> We match identifiers from a stratified random sample of approximately 420,000 loans (35,000 per year) out of 22 million. These 420,000 loans give 1.7 million loan-year observations. Initially, we estimate the model using CLTV constructed with a national HPI. Next, we compare actual and predicted rates for prepayment and default across time, size of city, and location within a city. To investigate the prediction errors, we repeat these exercises with CLTVs constructed using seven additional HPI measures, each with increasing geographic granularity. The model and empirical results are presented over the next several subsections.

### 3.1. Credit Model

The competing-options framework can be specified formally as a multinomial logit model with three outcomes,  $i = \{0, 1, 2\}$ , where a borrower remains current, prepays, or defaults on a mortgage.<sup>26</sup> The probability of a borrower choosing an outcome in each period,  $t$ , is

$$Pr(d_{jtz} = i) = \frac{\exp(X'_{jtz}\beta_i)}{1 + \sum_{\tau=1}^2 \exp(X'_{jtz}\beta_\tau)} \quad (8)$$

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<sup>25</sup>Public loan origination and performance data are becoming increasingly available. Both Freddie Mac and Fannie Mae provide single-family loan-level data going back until 2000 for more than 40 million loans. Freddie Mac data are available at [http://www.freddie.com/news/finance/sf\\_loanlevel\\_dataset.html](http://www.freddie.com/news/finance/sf_loanlevel_dataset.html). The Fannie Mae data can be found at <http://www.fanniemae.com/portal/funding-the-market/data/loan-performance-data.html>. The public datasets, though, do not have enough information to reliably identify the property location of individual loans. We obtain a proprietary identifier from Freddie Mac that allows us to match mortgages across physical addresses and merge information on house price transactions onto the loan performance data.

<sup>26</sup>Other outcomes, like repurchases and proprietary modifications, are possible and do exist in the actual loss sample data that we utilize. However, we focus on borrower behavior and, as is standard practice, we remove terminations that might be triggered by lenders, servicers, or investors.

Our contribution is analyzing how different measures of *CLTV* affect predicted outcomes.<sup>27</sup> Korteweg and Sorensen (2016) show that current approaches to updating LTV, including the use of city-level indices, can seriously understate the number of houses that are underwater by up to 80%, which has been shown to significantly increase the likelihood of default. Yang, Lin, and Cho (2011) find that not fully accounting for the cross-sectional dispersion of house prices is a major source of this bias and can significantly affect assessments of mortgage default risk. Building upon this research, we are particularly interested in how model accuracy is affected by CLTVs constructed using different levels of HPI granularly.

Our explanatory variables include typical credit model covariates for origination characteristics (e.g., credit score, debt-to-income, loan purpose, number of borrowers), current performance (e.g., unpaid principal balance, loan age, refinance burnout, yield curve swap spread, and current loan-to-value), and fixed effects to control for unobserved heterogeneity (e.g. origination year cohort, state). To remain consistent with earlier sections, we utilize a 20% holdout sample to compare predicted and actual outcomes on observations that have not been used during model calibration.

### **3.2. Empirical Results**

Results for overall model fit are presented in Figure 2.<sup>28</sup> Panels (a), (b), and (c) are based upon a model using a national HPI to calculate CLTV. Panel (a) illustrates that predicted and actual rates (of prepayment and default) are relatively close across the last 15 years, suggesting relative parameter stability across the sample. There are minor deviations when house prices decline after the housing boom in the early 2000s and again, when prices fell more drastically, in 2008–2010. To investigate whether disaggregated HPIs might improve model fit, we examine prediction errors across large and small cities as well as regions within a city. Prediction errors for prepayments and the geographic patterns are relatively consistent, with center-city areas having larger prediction errors. In contrast, for defaults, actual and predicted rates show noticeable differences. Small cities have lower default prediction errors in center-city regions and larger errors in mid-city and suburban regions. In contrast, large cities are associated with greater prediction errors in center-city regions. These prediction errors may partially be a function of measurement error arising from updating CLTVs with

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<sup>27</sup>Given our focus on CLTV, we build and test a model following industry guidance (see Dunsky et al. (2014)). Based upon out-of-sample tests, model fit appears acceptable. By working with a relatively standard model, we can focus on CLTV measurement error and its broad effect on model accuracy.

<sup>28</sup>Because the focus is on improving model fit, individual estimation coefficients are not presented here.

the national HPI.<sup>29</sup> Theory suggests measurement error will result in attenuation bias in the CLTV variable, which we estimate in panels (d) and (e). The graphs show point estimates for CLTV odds ratios and their confidence intervals constructed with increasingly granular HPis. Panel (d) depicts that an increase in CLTV reduces the relative risk of prepayment but the odds ratio estimate associated with the national index is the smallest (closest to 1). Panel (e) shows the CLTV odds ratio estimates for the default equation. Again, the estimate associated with the national index is the smallest (closest to 1). The magnitude of these two sets of coefficients suggests that an accurate CLTV has a larger estimated effect on default outcomes. This suggests that reducing measurement error will result in a stronger relationship between CLTV and mortgage performance.

Why might we expect a localized house price measure to improve model fit? House price appreciation rate gradients have been flat in small cities since 1990, but these gradients have tended to steepen in large cities (Bogin et al., 2016). These gradients are captured by a national HPI, introducing aggregation bias into CLTV predictions, especially in large cities. These CLTV errors may harm default model performance. On the other hand, were CLTVs correlated with other variables in the model, explanatory power may be absorbed by other parameter estimates, resulting in little net effect.

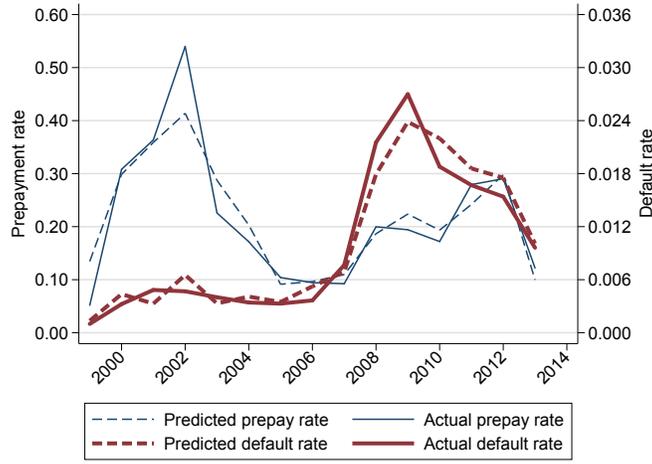
Our suite of HPis provide a variety of geographic aggregation levels to estimate the effect of HPI aggregation on mortgage performance models. To ensure an adequate sample size, we keep mortgages that fall within metropolitan areas that have at least 10,000 observations. Based on the property's location from the CBD, we compare how the choice of HPI granularity (i.e. a national HPI versus a metro HPI) affects model fit within concentric areas around the center of a city (e.g., center-city, mid-city, and suburbs). To evaluate model fit, we compute the RMSE and then consider whether it improves as estimations are performed with more granular HPis.

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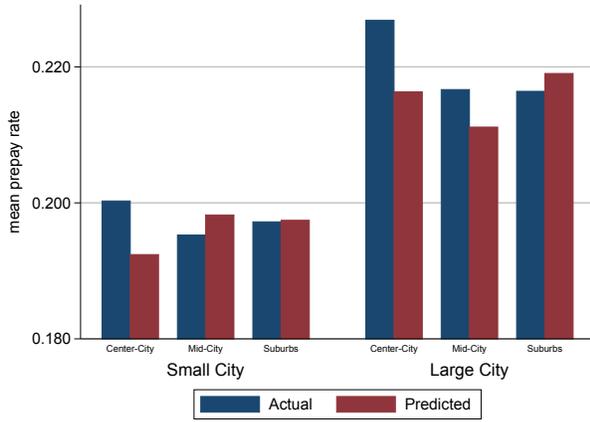
<sup>29</sup>A larger difference between actual and predicted rates in center-city regions of large cities should not be surprising. In such places, house price gradients are steeper and are increasing at a faster rate than in areas farther from the CBD. When a single HPI (like a national level) is used, the estimates will tend to underestimate price levels in center-city areas and over-estimate levels in outskirting areas. More granular HPis would reduce these errors. Borrowers will realize, too, that their actual house valuation is much higher (or CLTV is lower) and will be less likely to default than may be predicted with a national HPI. Similarly, since center-city areas appreciate at faster rates, refinancing may be more prevalent as borrowers repay to "lock-in" a more favorable rate when leverage is in their favor.

Figure 2: Estimates of Prepayment and Default Model

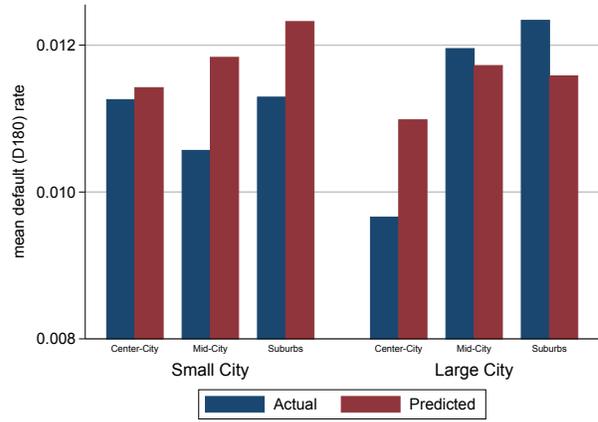
(a) Comparing Both Outcomes Across Performance Years



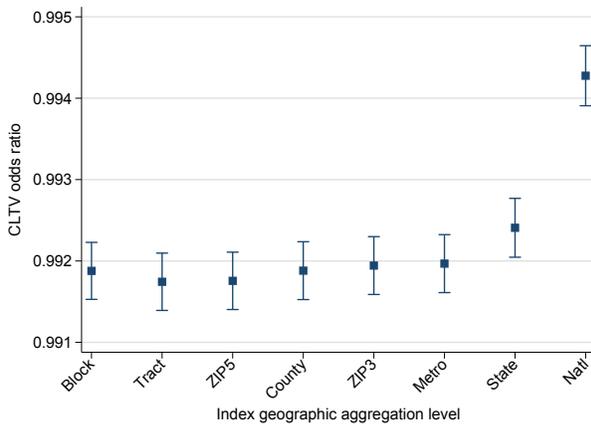
(b) Prepayments by Location



(c) Defaults by Location



(d) Prepayment CLTV Attenuation



(e) Default CLTV Attenuation

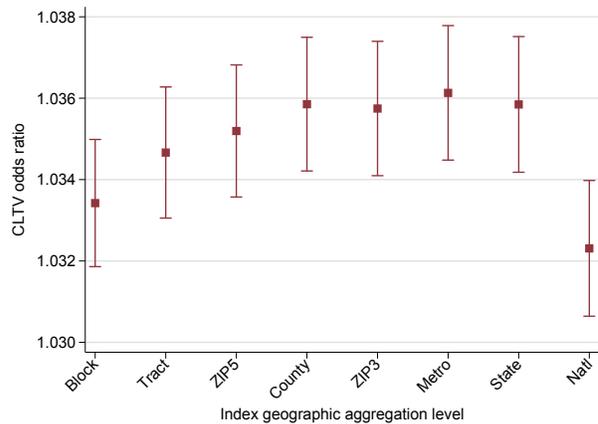
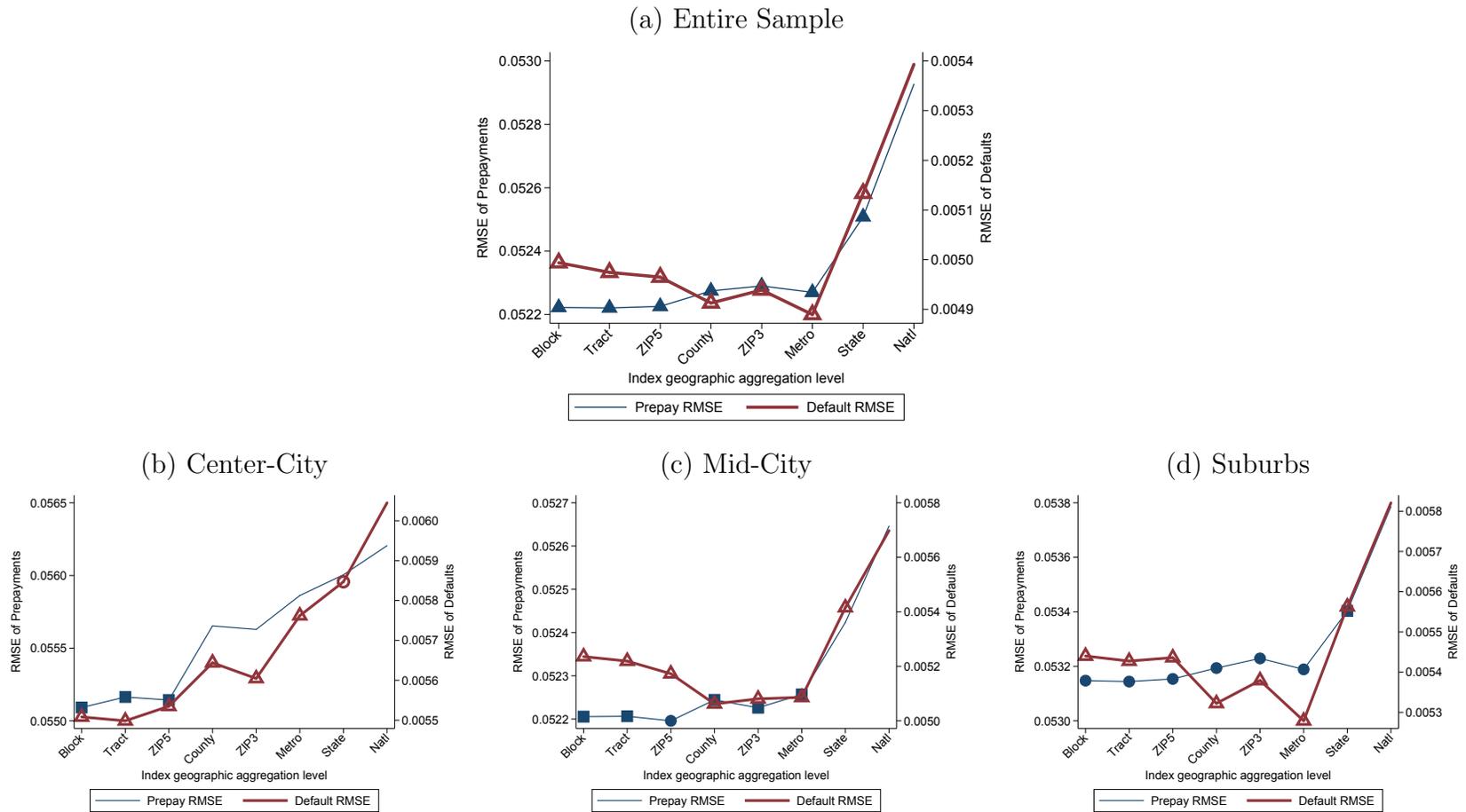


Figure 3: Comparing Fit by HPI Granularity and Region Within a City



**Note:** Mean values, of actual prepayments and defaults, and predictions are computed within performance year and for each region within a city that has at least 10,000 observations in our holdout sample (the four panels shown above). Statistical significance (per a  $F$ -distribution) is computed by taking the ratio of the RMSE of an HPI granularity level compared to the RMSE for the national HPI. Geometric shapes illustrate when a more granular HPI improves model fit (i.e., a triangle for  $p \leq 0.01$ , a circle for 0.05, and a square for 0.10).

Figure 3 shows RMSE decreasing as we transition from the national and state HPIs to lower levels of geography (from right to left). As illustrated, model fit is consistently poorest (i.e. highest RMSE) when a national HPI is used—no matter the location within the city. A fair question, though, is whether the differences are statistically significant. We use a variation of Theil’s  $U$ -statistic to determine whether a more granular HPI improves model fit.<sup>30</sup> Geometric shapes indicate a statistical difference (in order of significance: a triangle for  $p \leq 0.01$ , a circle for 0.05, and a square for 0.10).<sup>31</sup> In panel (a), the national HPI is outperformed by every other level of geography, for both prepayments and defaults, at the 1% level of significance. As done throughout this paper, we can assess the results further by analyzing regions within a city.

Several findings emerge in the bottom three panels. First, model fit is almost always improved with a more granular HPI for both prepayment and default estimations. Second, the most localized HPI does not always improve model fit the most. In a stepwise fashion, RMSE is improved by going down to at least a metro-level HPI but, with the exception of center-city areas, not much else is gained by using finer HPIs.<sup>32</sup> Third, in center-city areas, model fit is improved by using an even more local measure of house prices—at least a ZIP5 HPI.<sup>33</sup> Overall, the results suggest that credit model fit can be improved by moving down to at least a MSA HPI, and even further in centralized areas of larger cities where location matters most.

## 4. Conclusion

The last decade’s financial crisis illustrated the danger of not fully understanding the fundamental characteristics of the assets underlying complex financial instruments. For mortgage

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<sup>30</sup>The statistic is the ratio of the RMSE of the national HPI with the RMSE for another level of HPI granularity. Recall that a  $\chi^2$  distribution is the sum of squares of statistically independent Gaussian variables and that the ratio of two  $\chi^2$  distributions is an  $F$ -distribution. We use the sample size of each group as the degrees of freedom and critical  $p$ -values are calculated separately for each panel but are the same for each choice of HPI granularity within a panel. For example, the critical values for ratios in the center-city are 1.0375 for the 10% level of significance, 1.0484 for 5%, and 1.0691 for the 1% level.

<sup>31</sup>Prepayment outcomes have solid shapes and default outcomes have hollow ones.

<sup>32</sup>This comment is made from the perspective of moving down in one-level increments (where moving from a metro to a ZIP3 or lower level might not result in a statistical improvement of RMSE). Although a state HPI usually dominates a national HPI, the metro-level HPI also dominates the state HPI when we perform additional  $F$ -tests. Outside of center-city areas, the metro HPI appears to offer a sufficient level of granularity for prepay and default modeling. Model fit does not usually improve, in a statistical sense, from using a more granular index beyond those levels. An exception is the center-city region where RMSE is continually improved until the ZIP5 HPI and further disaggregation is not necessary.

<sup>33</sup>This reiterates an earlier finding: no single index can best reduce model fit across all areas. Rather, to optimize fit, it might be best to use a combination of indices.

backed securities, an important feature of the mortgage collateral is the location of the properties in the reference pool. In this paper, we offer new evidence that the geographic scope of house price indices can affect price valuations and mortgage performance modeling for such collateral.

We find that local constant-quality HPIs capture within-city heterogeneity of house price levels and appreciation rates, which can compound over time, particularly in the center-cities of large cities. This known heterogeneity means that less granular indices (i.e., ZIP3, state, or national HPIs) can lead to market value errors that are predictable spatially, as a function of distance to the central business district (CBD), and the errors can be non-trivial. The consequences of geographic aggregation hinge on the relative steepness of house price gradients within an area: in large cities with steep gradients, aggregation has substantial costs, but in smaller or more supply-elastic cities, aggregation is relatively benign.

We also find that CLTVs estimated using local HPIs result in lower prepayment and default rate model errors relative to models using CLTVs estimated using state or national HPIs. The results suggest that CLTVs constructed using more aggregated HPIs tend to capture general trends in borrower equity, but with substantial remaining error. The benefits of disaggregation are most striking near the centers of large cities, where performance is substantially improved by estimating CLTVs using a ZIP code index.<sup>34</sup>

Overall, the realtors' mantra of "location, location, location" is apparently true as location seems to have important implications for housing finance. A mortgage's value at origination and ongoing loan performance are often more accurately measured with a local HPI. Our analysis suggests this is particularly true in larger cities, where results are most sensitive to the choice of a house price index and the effect of "missing the mark".

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<sup>34</sup>Although beyond the scope of this paper, this within-city evidence of local house price appreciation has implications for risk-return tradeoffs. Han (2013) finds cities with inelastic housing and growing populations tend to have strong hedging incentives. Our results suggest these effects might be further magnified in centralized areas within larger cities.

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